[09-02-11-T10]

Square root of 2 irrational

◆ Def. (Irrational) A number x is irrational if $x \neq \frac{p}{q}$, $p, q \in \mathbb{Z}$.

■ **The square root of *2* is an irrational number**

Proof (indirect)

Suppose that $\sqrt{2}$ is a rational number. Then, $\sqrt{2} = \frac{p}{q}$, $p, q \in \mathbb{Z}$, where $\frac{p}{q}$ is written in most reduced form; that is, there is no factor common to both p and q. Now,

$$\sqrt{2} = \frac{p}{q} \Longrightarrow 2 = \frac{p^2}{q^2} \Longrightarrow 2 q^2 = p^2$$

But, this means p^2 is an even number. Since the square root of an even number is an even number, p is even. Thus we may write p as $2k$, $k \in \mathbb{Z}$. Then, $p^2 = 4k^2 = 2q^2$, which implies that $q^2 = 2k^2$. Thus, q is an even number which we may write as $2r$, $r \in \mathbb{Z}$. Then,

$$\frac{p}{q} = \frac{2k}{2r}$$

which shows us that p and q do have a common factor, 2. This, however, contradicts our condition that p,q have no common factor. So, the supposition that $\sqrt{2}$ is rational is false.

∴ $\sqrt{2}$ is an irrational number.

□